

Social Network Analysis: Centrality Measures

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What is centrality? I

- Centrality measures address the question:
"Who is the most important or central person in this network?"
- There are many answers to this question, depending on what we mean by importance.
- According to Scott Adams, the power a person holds in the organization is inversely proportional to the number of keys on his keyring.
 - A janitor has keys to every office, and no power.
 - The CEO does not need a key: people always open the door for him.
- There are a vast number of different centrality measures that have been proposed over the years.

Degree centrality for undirected graph I

- The nodes with higher degree is more central.
- Let $A \in \mathbb{R}^{n \times n}$ be the adjacency matrix of a undirected graph. Let $k \in \mathbb{R}^n$ be the degree vector. Let $e \in \mathbb{R}^n$ be the all-one vector. Then

$$k = Ae$$

- For comparison purpose, we can standardize the degree by dividing by the maximum possible value $n - 1$.
 - Degree is simply the number of nodes at distance one.
- Though simple, degree is often a highly effective measure of the influence or importance of a node:
 - In many social settings people with more connections tend to have more power and more visible.

Closeness centrality for undirected graph

- The farness/peripherality of a node v is defined as the sum of its distances to all other nodes
- The closeness is defined as the inverse of the farness.

$$closeness(v) = \frac{1}{\sum_{i \neq v} d_{vi}}$$

- For comparison purpose, we can standardize the closeness by dividing by the maximum possible value $1/(n-1)$
- If there is no (directed) path between vertex v and i then the total number of vertices is used in the formula instead of the path length.
- The more central a node is, the lower its total distance to all other nodes.
- Closeness can be regarded as a measure of how long it will take to spread information from v to all other nodes sequentially.

Betweenness centrality

- Betweenness centrality quantifies the number of times a node acts as a bridge along the shortest path between two other nodes.
- It was introduced as a measure for quantifying the control of a human on the communication between other humans in a social network by Linton Freeman.
- In this conception, vertices that have a high probability to occur on a randomly chosen shortest path between two randomly chosen vertices have a high betweenness.