Data Analysis and Statistical Software

Introduction to Bayesian Analysis Using Stata

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Outline

- Introduction to Bayesian Analysis
- Coin Toss Example
- Priors, Likelihoods, and Posteriors
- Markov Chain Monte Carlo (MCMC)
- Bayesian Linear Regression
- Advantages and Disadvantages of Bayes

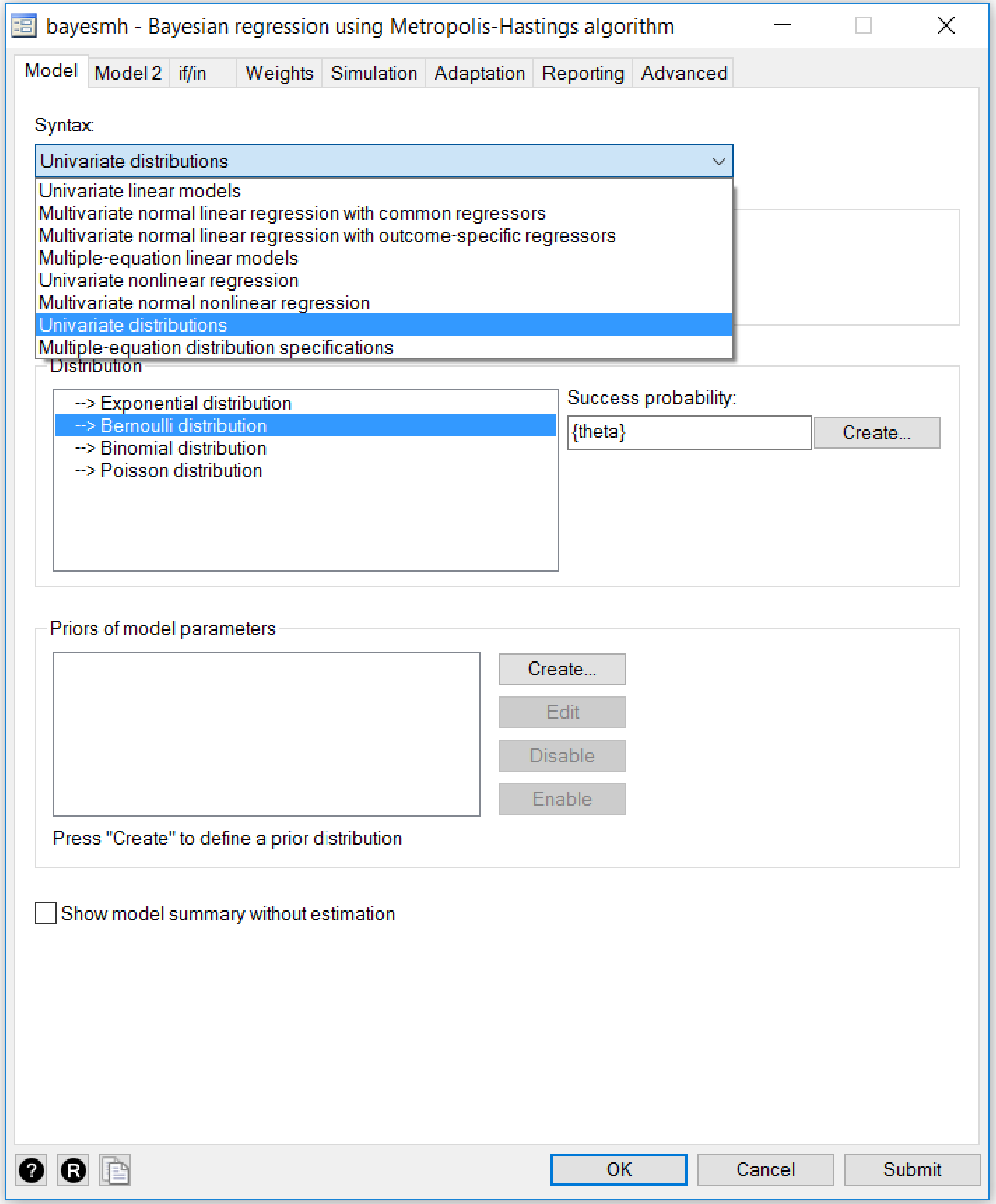
STATA BAYESIAN ANALYSIS REFERENCE MANUAL RELEASE 14

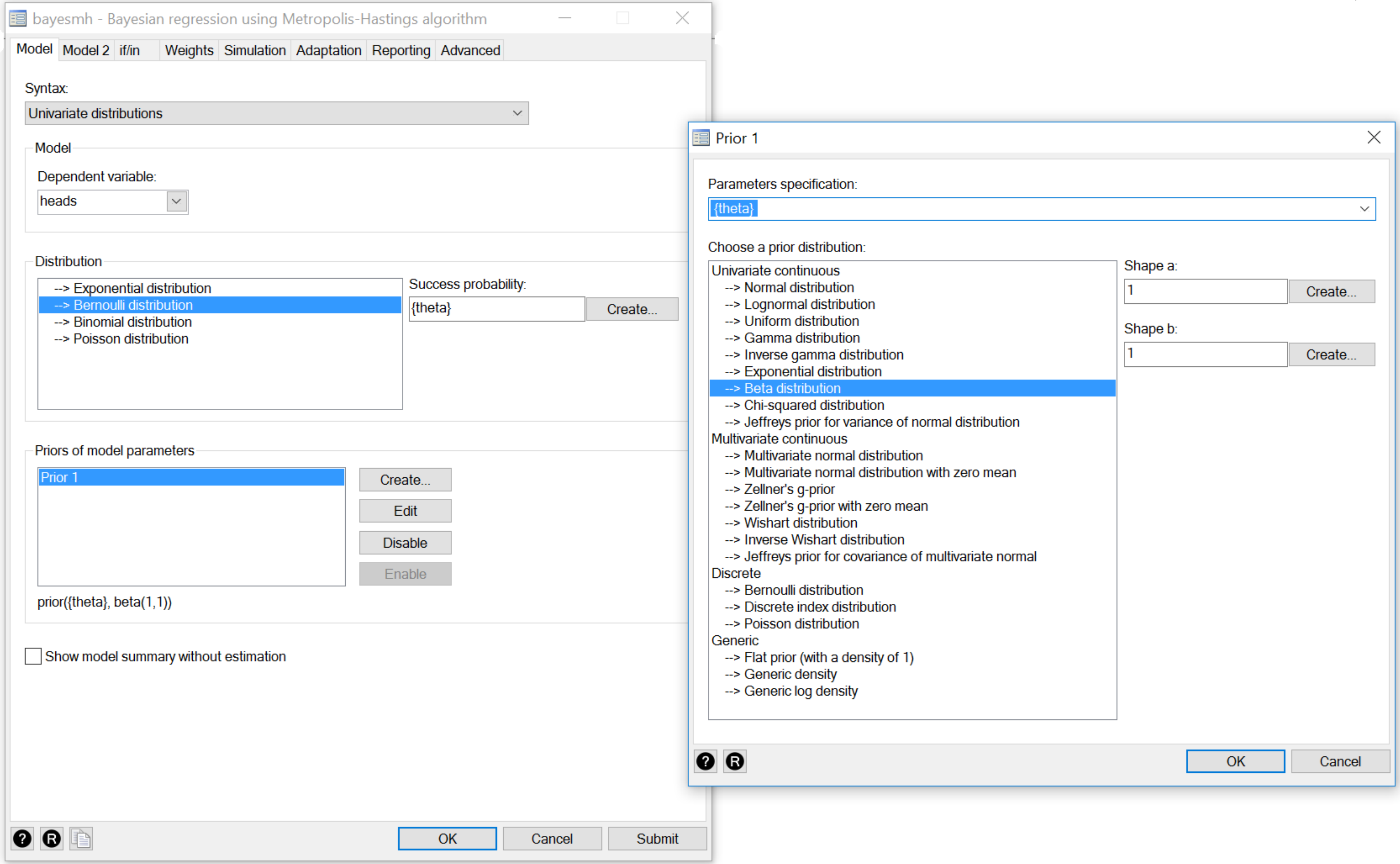
Title

bayesmh — Bayesian regression using Metropolis–Hastings algorithm

Description

bayesmh fits a variety of Bayesian models using an adaptive Metropolis–Hastings (MH) algorithm. It provides various likelihood models and prior distributions for you to choose from. Likelihood models include univariate normal linear and nonlinear regressions, multivariate normal linear and nonlinear regressions, generalized linear models such as logit and Poisson regressions, and multiple-equations linear models. Prior distributions include continuous distributions such as uniform, Jeffreys, normal, gamma, multivariate normal, and Wishart and discrete distributions such as Bernoulli and Poisson. You can also program your own Bayesian models; see [BAYES] **bayesmh evaluators**.


bayesmh - Bayesian regression using Metropolis-Hastings algorithm
Model
Model 2
if/in
Weights
Simulation
Adaptation
Reporting
Advanced
Syntax:
Univariate distributions
Univariate linear models
Multivariate normal linear regression with common regressors
Multivariate normal linear regression with outcome-specific regressors
Multiple-equation linear models
Univariate nonlinear regression
Multivariate normal nonlinear regression
Univariate distributions
Multiple-equation distribution specifications
Distribution
--> Exponential distribution
--> Bernoulli distribution
--> Binomial distribution
--> Poisson distribution
Success probability:
{theta}
Create...
Priors of model parameters
Create...
Edit
Disable
Enable
Press "Create" to define a prior distribution
Show model summary without estimation
OK
Cancel
Submit


bayesmh - Bayesian regression using Metropolis-Hastings algorithm
Model
Model 2
if/in
Weights
Simulation
Adaptation
Reporting
Advanced
Syntax:
Univariate distributions
Model
Dependent variable:
heads
Distribution
--> Exponential distribution
--> Bernoulli distribution
--> Binomial distribution
--> Poisson distribution
Success probability:
{theta}
Create...
Priors of model parameters
Prior 1
Create...
Edit
Disable
Enable
prior({theta}, beta(1,1))
Show model summary without estimation
OK
Cancel
Submit
Prior 1
Parameters specification:
{theta}
Choose a prior distribution:
Univariate continuous
--> Normal distribution
--> Lognormal distribution
--> Uniform distribution
--> Gamma distribution
--> Inverse gamma distribution
--> Exponential distribution
--> Beta distribution
--> Chi-squared distribution
--> Jeffreys prior for variance of normal distribution
Multivariate continuous
--> Multivariate normal distribution
--> Multivariate normal distribution with zero mean
--> Zellner's g-prior
--> Zellner's g-prior with zero mean
--> Wishart distribution
--> Inverse Wishart distribution
--> Jeffreys prior for covariance of multivariate normal
Discrete
--> Bernoulli distribution
--> Discrete index distribution
--> Poisson distribution
Generic
--> Flat prior (with a density of 1)
--> Generic density
--> Generic log density
Shape a:
1
Create...
Shape b:
1
Create...
OK
Cancel

The **bayesmh** Command

```
bayesmh sbp age sex bmi,                          ///
   likelihood(normal({sigma2}))                   ///
   prior({sbp: _cons}, normal(0,100))             ///
   prior({sbp: age},   normal(0,100))             ///
   prior({sbp: sex},   normal(0,100))             ///
   prior({sbp: bmi},   normal(0,100))             ///
   prior({sigma2},      igamma(1,1))
```

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Title

bayes — Bayesian regression models using the bayes prefix

Description

The bayes prefix fits Bayesian regression models. It provides Bayesian support for many likelihood-based estimation commands. The bayes prefix uses default or user-supplied priors for model parameters and estimates parameters using MCMC by drawing simulation samples from the corresponding posterior model. Also see [BAYES] **bayesmh** and [BAYES] **bayesmh evaluators** for fitting more general Bayesian models.

The **bayes** Prefix

```
regress sbp age sex
bayes: regress sbp age sex

logistic highbp age sex
bayes: logistic highbp age sex
```

Two Paradigms

Frequentist Statistics

Model parameters are considered to be unknown but fixed constants and the observed data are viewed as a repeatable random sample.

Bayesian Statistics

Model parameters are random quantities which have a posterior distribution formed by combining prior knowledge about parameters with the evidence from the observed data sample.

Reverend Thomas Bayes

- 1701 – born in London
- Presbyterian Minister
- Amateur Mathematician
- Published one paper on theology and one on mathematics
- 1761 – died in Kent
- 1763 - “Bayes Theorem” paper published by friend Richard Price

https://bayesian.org/bayes

Coin Toss Example

What is the probability of heads (θ)?

Prior Distribution

Prior distributions are probability distributions of model parameters based on some a priori knowledge about the parameters.

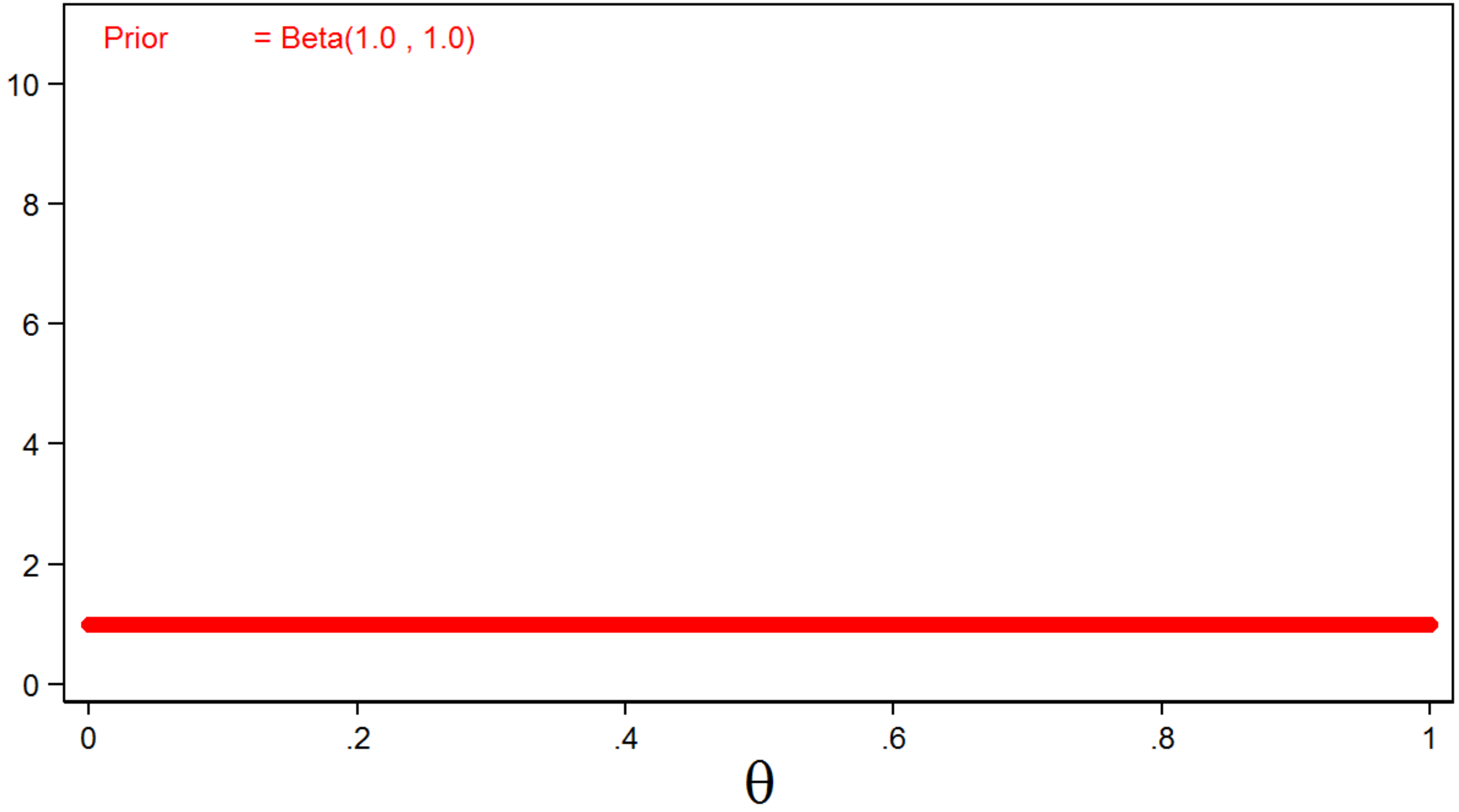
Prior distributions are independent of the observed data.

Beta Prior for θ

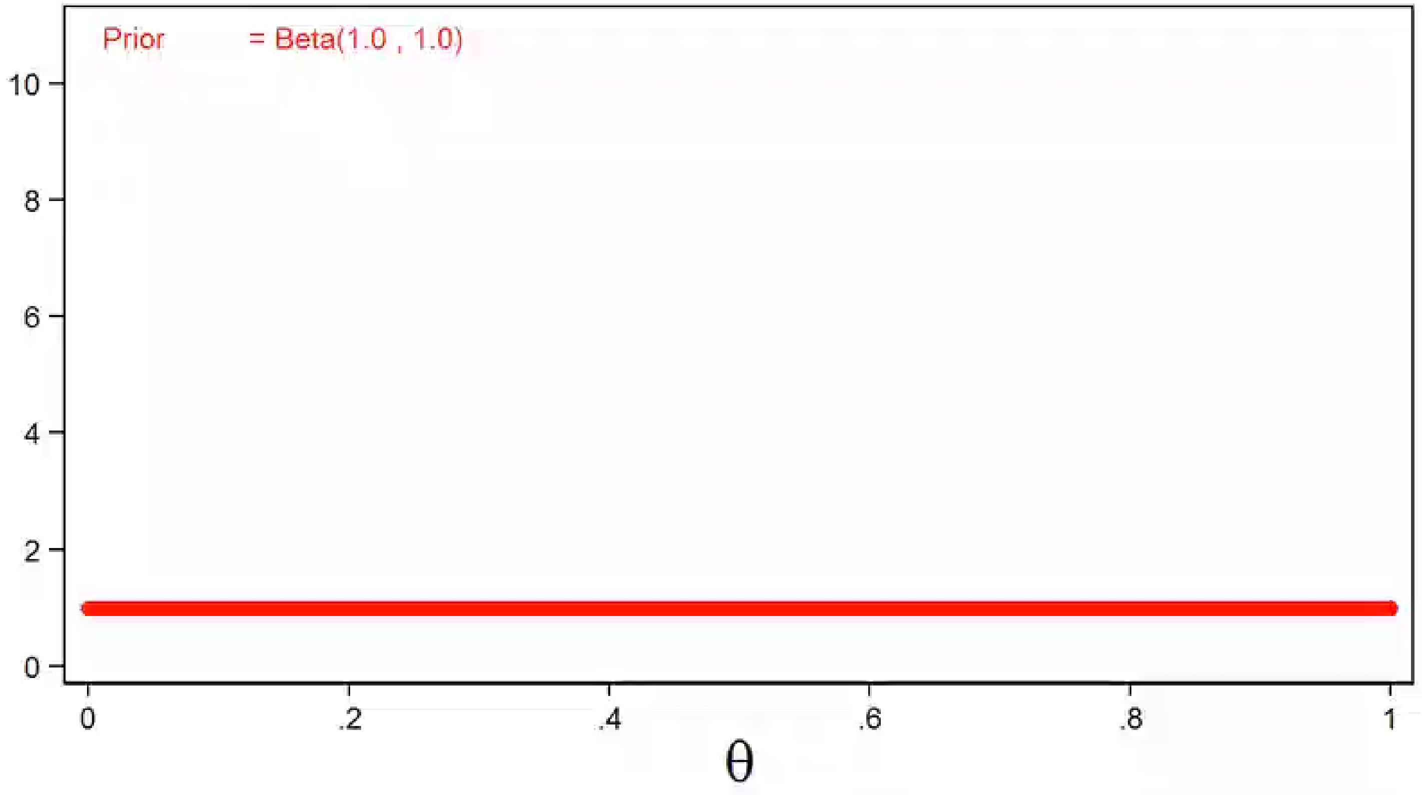
$$P(\theta) = Beta(\alpha, \beta)$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{(\alpha-1)}(1-\theta)^{(\beta-1)}$$

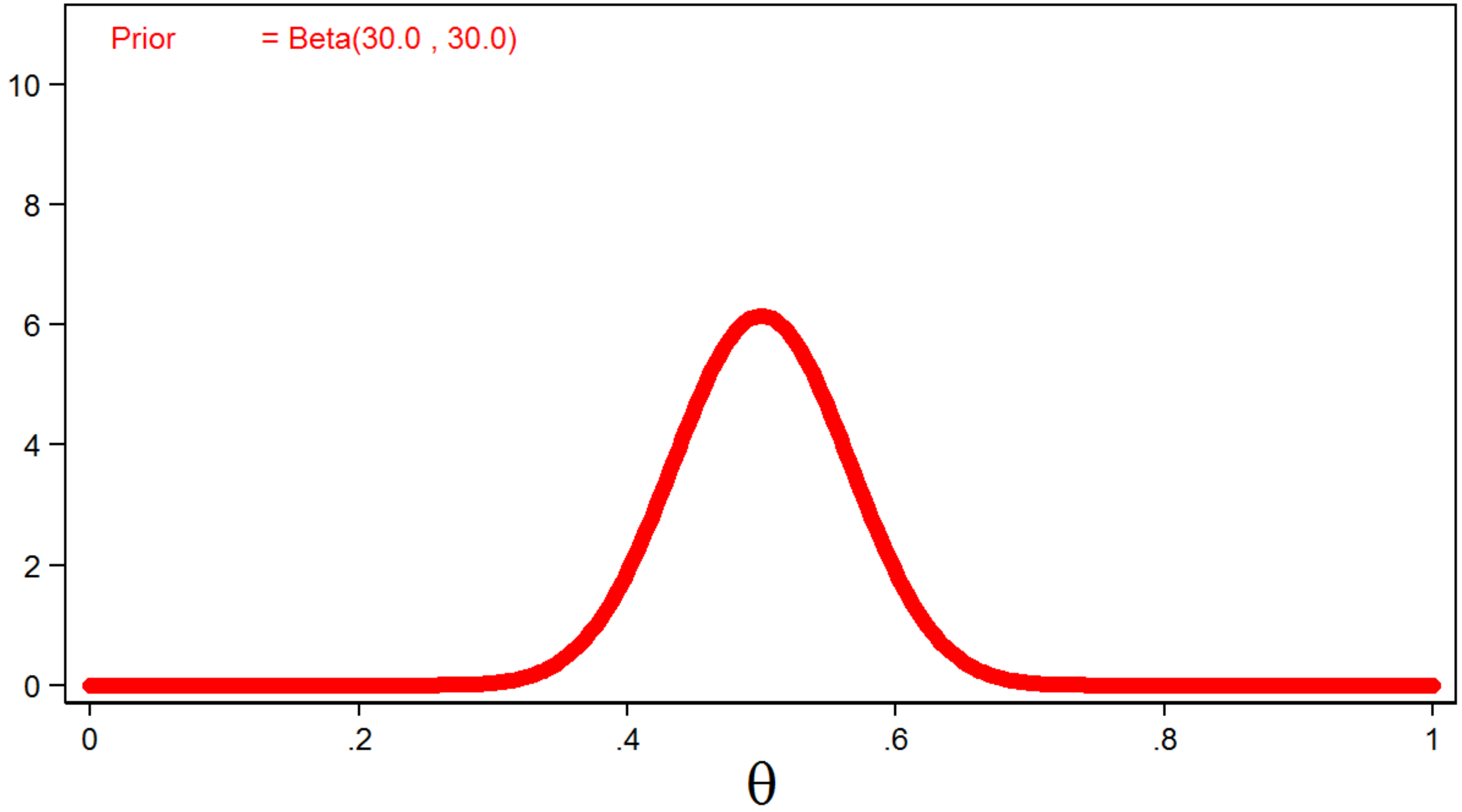
Uninformative Prior



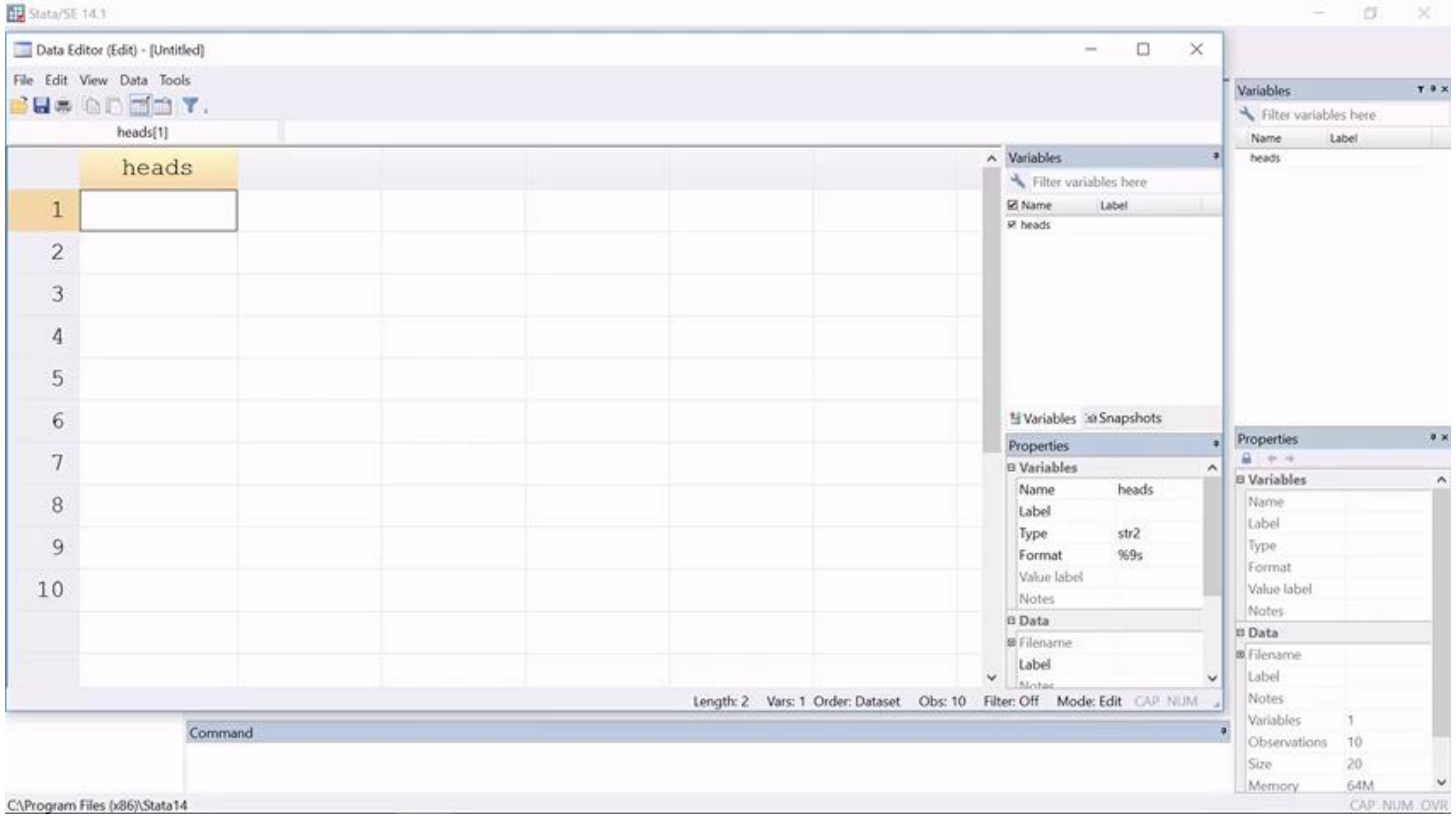
Different Priors



Informative Prior



Coin Toss Experiment

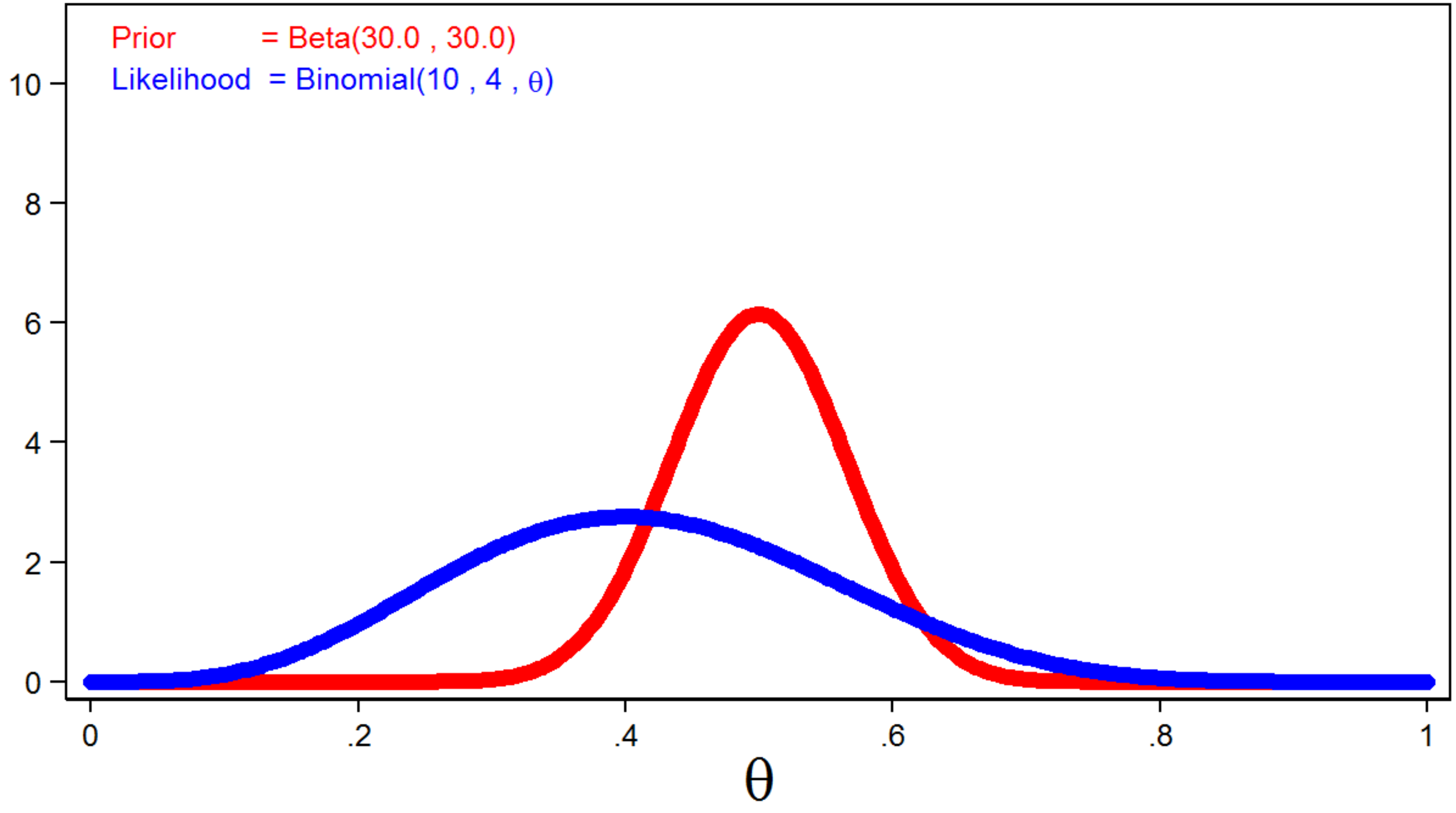


Likelihood Function for the Data

$$P(y|\theta) = Binomial(n, \theta)$$

$$= \binom{n}{y} \theta^{y} (1 - \theta)^{(n-y)}$$

Prior and Likelihood



Posterior Distribution

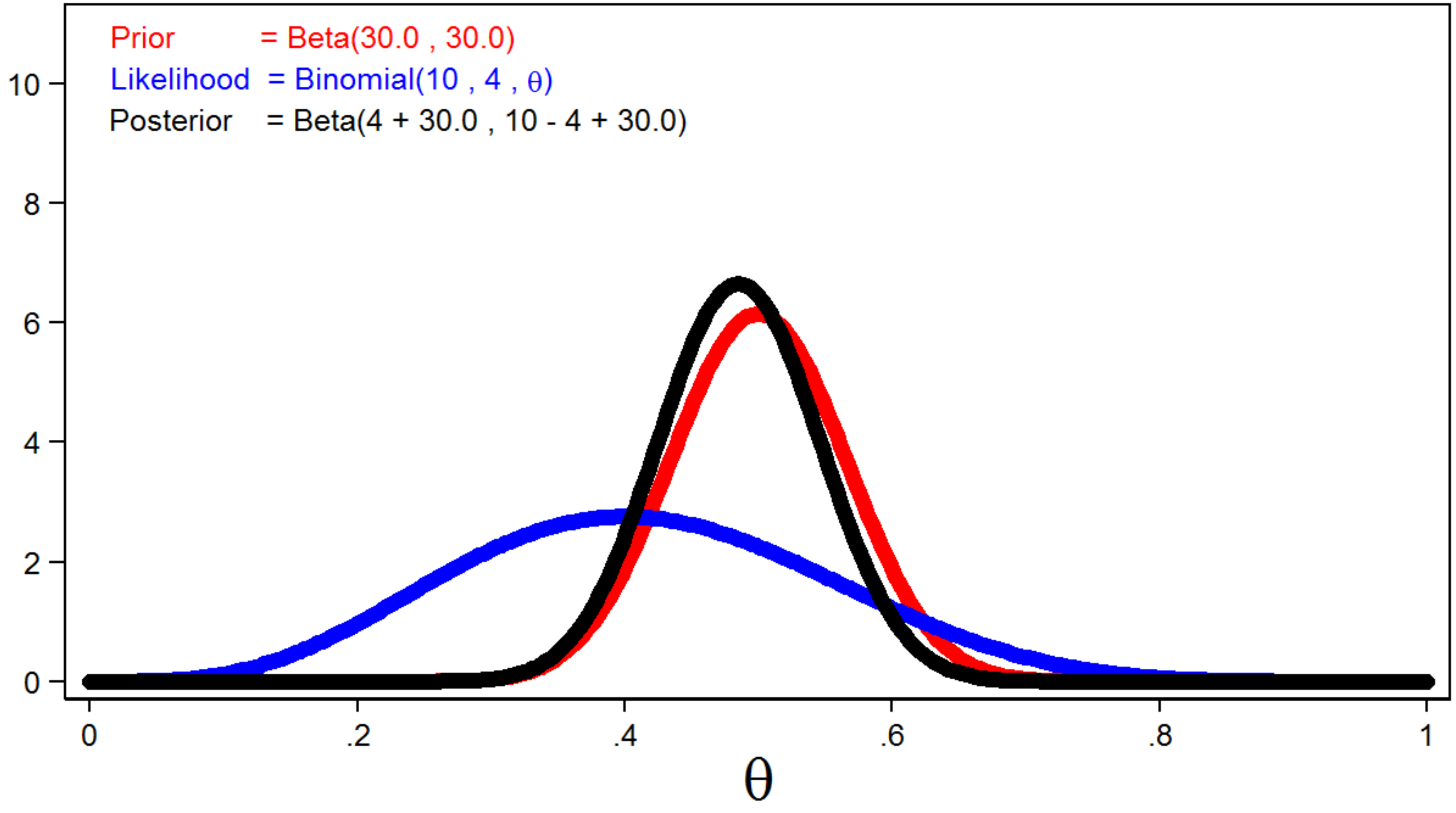
$$Posterior = Prior \times Likelihood$$

$$P(\theta|y) = P(\theta)P(y|\theta)$$

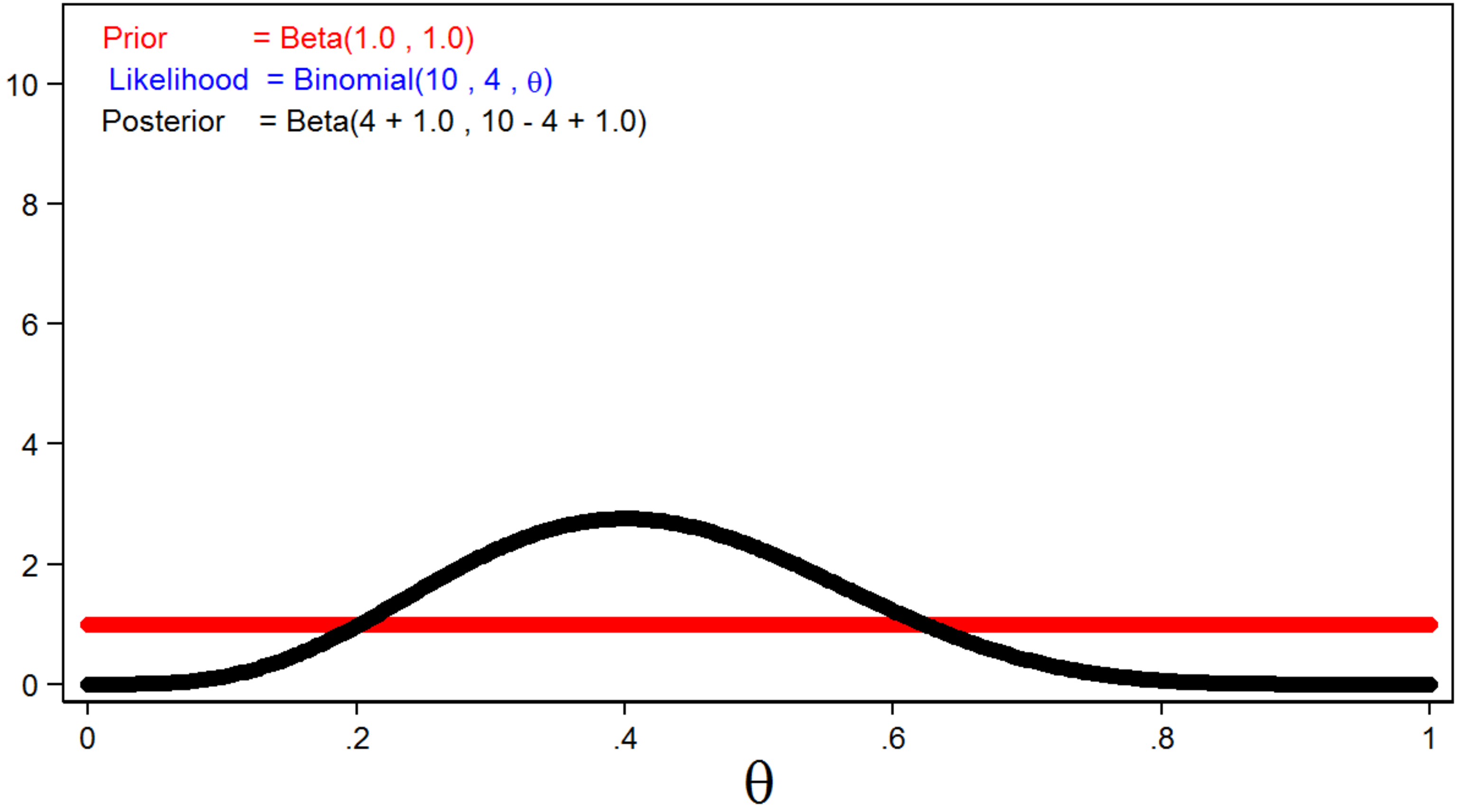
$$P(\theta|y) = Beta(\alpha, \beta) \times Binomial(n, \theta)$$

$$= Beta(y + \alpha, n - y + \beta)$$

Posterior Distribution



Effect of Uninformative Prior



Effect of Informative Prior